**Introduction**

**Main Object:** Random Walks in Changing Environments (RWCEs): At vertex $x$, the RWCE jumps to a neighboring vertex $y$ with probabilities proportional to edge-weights. Moreover, the edge-weights are random variables changing after each timestep.

**Definition 1 (RWCE):** A Random Walk in Changing Environments on a graph $G = (V,E)$ is a stochastic process $\{(X_t, w_t)\}_{t=0}^{\infty}$ such that for any $y \in V$, we have

$$P(X_{t+1} = y \mid X_t) = \frac{w_t(X_t, y)}{w_t(X_t, V)}$$

where $X_t \in V$, $w_t \in W_G$, and $F_t = \sigma(X_0, \ldots, X_t, w_0, \ldots, w_t)$ for each $t \geq 0$.

**Central Question.** Assume that $(G, w)$ is a.s. recurrent (resp. transient) for each $t \geq 0$. When is $(X_t, w_t)_{t=0}^{\infty}$ also recurrent (resp. transient)?

**Definition 2 (Recurrence/Transience):** An RWCE on $G = (V,E)$ is recurrent if every vertex is visited infinitely often almost surely. It is transient if every vertex is visited finitely often almost surely. Otherwise, the RWCE is of mixed-type.

The following example shows that nonelliptic RWCEs can be neither recurrent nor transient.

![Fig. 1: An Infinite Graph](image)

**Steps 1 and 2.** We reduce the problem to the special case where $s$ has a single neighbor.

**Steps 3 and 4.** We study recurrence by constructing a supermartingale and using the optional stopping theorem.

**Graphs and Electrical Networks**

Assume that some $s \in V$ is fixed as the origin.

**Definition 4 (Voltages):** Let $V := \{v \in V : d(s,v) \leq n\}$ and $\partial V := \{v \in V : d(s,v) = n\}$. For $n \geq 1$, $t \geq 0$, and $x \in V$, let $v_n(x)$ denote the (random) voltage of $x$ in $(G, w_n)$ when $s$ is grounded and $\partial V$ is kept at voltage 1.

The key connection between random walks on graphs and electrical networks is that $v_n(x)$ equals the probability that a random walk on $(G, w_n)$ beginning at $x$ will hit $\partial V$ before $s$. In particular, both quantities are harmonic, meaning that $v_n(x)$ is a weighted average of $v_{n+1}(y)$ where $y \sim x$.

**Definition 5 (Effective Resistance):** The effective resistance of $(G_n, w_n)$ with $s$ given equals

$$R_{eff,n} := \sum_{e \in E} v_n(e)^2$$

where $i$ is the amount of flow through $e$.

Then, a random walk on $(G, w)$ is recurrent if and only if

$$\lim_{n \to \infty} R_{eff,n} = \infty.$$